

Temporally multiplexed quantum repeaters with atomic gases

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We propose a temporally multiplexed version of the Duan-Lukin-Cirac-Zoller (DLCZ) quantum repeater protocol using controlled inhomogeneous spin broadening in atomic gases. A first analysis suggests that the advantage of multiplexing is negated by noise due to spin wave excitations corresponding to unobserved directions of Stokes photon emission. However, this problem can be overcome with the help of a moderate-finesse cavity which is in resonance with Stokes photons, but invisible to the anti-Stokes photons. Our proposal promises greatly enhanced quantum repeater performance with atomic gases.

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The distribution of entanglement over long distances is an interesting challenge both for fundamental reasons and for applications such as quantum key distribution and future quantum networks. It is a difficult task because of transmission losses, for example 1000 km of optical fiber have a transmission of 10^{-20} . Conventional amplification as in classical telecommunications is ruled out by the no-cloning theorem [1]. A possible solution is the use of quantum repeaters [2], which are based on creating and storing entanglement in moderate-distance elementary links and then extending it by entanglement swapping. The DLCZ proposal [3] for realizing quantum repeaters has inspired many experiments [4–8]. It is based on ensembles of three-level systems, typically atomic gases. The spontaneous Raman emission of a photon, which we will call the *Stokes* photon, creates a heralded single atomic excitation in the ensemble. The detection of a photon that could have come from either of the two ensembles, in a way that erases all which-way information, leads to an atomic excitation that is in a coherent superposition state of being in either of the two ensembles, cf. Fig. 1 of Ref. [3]. This is an entangled state, which forms the elementary link in the DLCZ protocol. The entanglement can be extended by re-converting the atomic excitations into *anti-Stokes* photons and detecting them in the same way as the Stokes photons.

The DLCZ protocol is attractive because it uses quite simple ingredients. Unfortunately it is too slow to be practical, even under optimistic assumptions for re-conversion and detection efficiencies, storage times etc. This recognition has recently led to proposals for improvements [8–11]. The most significant improvements can be achieved through the use of multiplexing [8, 10–12]. We recently proposed an attractive form of *temporal* multiplexing that combines photon pair sources and quantum memories that can store many temporal modes [11]. Solid-state atomic ensembles are well suited for realizing such temporal multi-mode memories [11, 13–16].

Nevertheless, given that the most advanced experiments on quantum repeaters so far have been performed with atomic gases [6, 7], it is also of great interest to search for ways of achieving temporal multiplexing in such systems. In the temporal multi-mode memories of Refs. [11, 13–16], photons are stored on the optical transition. The multi-mode character is achieved thanks to the static inhomogeneous broadening of that transition, in combination with photon echo techniques. In the DLCZ protocol, which is based on Raman emission, information is stored as a spin excitation. It is then natural to look for a multi-mode protocol that involves inhomogeneous broadening of the spin transition in combination with spin echos. For other memory protocols relying on spin echos see Refs. [17, 18].

In the DLCZ protocol, we are dealing with a large number N_A of Lambda atoms with two ground state levels g and s and an excited state level e , cf. Fig. 1. Initially all atoms are in g . The Raman emission of the Stokes photon creates a state $|\psi\rangle = \frac{1}{\sqrt{N_A}}(e^{i(\mathbf{k}_w - \mathbf{k}_S) \cdot \mathbf{x}_1}|s\rangle_1|g\rangle_2 \dots |g\rangle_{N_A} + \dots + e^{i(\mathbf{k}_w - \mathbf{k}_S) \cdot \mathbf{x}_{N_A}}|g\rangle_{N_A-1}|s\rangle_{N_A})$, where \mathbf{x}_n is the position of the n -th atom, \mathbf{k}_w is the \mathbf{k} vector of the write laser beam, which is slightly detuned from the $g - e$ transition, and \mathbf{k}_S that of the Stokes photon, which is emitted on the $e - s$ transition. The state $|\psi\rangle$ describes a single *spin wave* excitation with \mathbf{k} vector $\mathbf{k}_w - \mathbf{k}_S$. Applying a read laser with $\mathbf{k}_r = -\mathbf{k}_w$ on the $s - e$ transition will lead to the emission of an anti-Stokes photon, associated with the return to the atomic initial state $|g\rangle_1 \dots |g\rangle_{N_A}$. For the state $|\psi\rangle$ the amplitude for the emission of the anti-Stokes photon in direction \mathbf{k}_{AS} is proportional to $\sum_{n=1}^{N_A} e^{-i(\mathbf{k}_{AS} + \mathbf{k}_S) \cdot \mathbf{x}_n}$, which for a large ensemble consisting of many atoms is strongly peaked around $\mathbf{k}_{AS} = -\mathbf{k}_S$.

Now consider the case where there is inhomogeneous spin broadening. This means that different atoms have slightly different energy separations between g and s , where for each atom we will denote the detuning from the center frequency of the $g - s$ transition by

ω_n . In this case, if a time t has elapsed since the Stokes emission, the above state $|\psi\rangle$ has to be replaced by $|\tilde{\psi}\rangle = \frac{1}{\sqrt{N_A}}(e^{-i\omega_1 t} e^{i(\mathbf{k}_w - \mathbf{k}_s) \cdot \mathbf{x}_1} |s\rangle_1 |g\rangle_2 \dots |g\rangle_{N_A} + \dots + e^{-i\omega_{N_A} t} e^{i(\mathbf{k}_w - \mathbf{k}_s) \cdot \mathbf{x}_{N_A}} |g\rangle_1 \dots |g\rangle_{N_A-1} |s\rangle_{N_A})$, and the anti-Stokes photon emission amplitude is now proportional to $\sum_{n=1}^{N_A} e^{-i\omega_n t} e^{-i(\mathbf{k}_{AS} + \mathbf{k}_s) \cdot \mathbf{x}_n}$, which in general no longer has a peak for any direction of \mathbf{k}_{AS} , due to the temporal phase factors $e^{-i\omega_n t}$ that vary from atom to atom. However, suppose that we are capable of flipping the sign of the atomic detunings, $\omega_n \rightarrow -\omega_n$, for all n [13, 14, 17, 18]. This requires the inhomogeneous broadening to be controllable, for example it could be due to a magnetic field gradient, cf. below. The sign can then be flipped by flipping the sign of the magnetic field [17, 18]. In this case, after another time t all the temporal phases have canceled, i.e. the state $|\tilde{\psi}\rangle$ has evolved back into $|\psi\rangle$. If the read pulse is applied now, the anti-Stokes emission will again be highly directional. If no read pulse is applied, the spin wave will simply dephase again.

We are now ready to describe the basic idea of our proposed multi-mode protocol. For large N_A and in the absence of atom-atom interactions, spin waves corresponding to different emission times and directions are completely independent. Consider spin waves created at times t_1, t_2, t_3 etc. If the atomic detunings are switched at a time T , then these spin waves rephase at times $2T - t_1, 2T - t_2, 2T - t_3$ etc. By applying the read laser at a specific time, one can ensure that only one specific spin wave is in phase. Entanglement creation in the original DLCZ protocol involves many unsuccessful attempts. Each application of the write laser triggers the emission of a Stokes photon in a given direction only with a small probability (it has to be kept small because of multi-excitation errors, see below). Most of the Stokes photons will moreover be lost in long-distance transmission. After every write pulse one has to wait for information whether the photon was detected at the far-away central station between the two repeater nodes under consideration. For a typical distance between nodes of $L_0 = 100$ km, the waiting time is $L_0/c = 500\mu\text{s}$, taking into account the reduced speed of light in optical fibers. This leads to low repetition rates, and thus to very low entanglement creation rates. In contrast, Ref. [11] showed that the capability to perform multi-mode storage and selective recall allows one to apply the write laser many times in quick succession. One can subsequently read out exactly that spin wave for which the entanglement generation was in fact successful and use it for entanglement swapping etc. The present approach seems to promise a greatly improved quantum repeater rate based on this principle, see also sec. VI.A of Ref. [19].

Unfortunately the described protocol has a serious problem, which is absent for the scheme of Ref. [11]. In typical experiments one detects only those Stokes pho-

tons that are emitted in one specific direction, which is defined by the geometry of the experiment. However, the Stokes emission process itself is completely non-directional (following a dipole emission pattern). This means that most Stokes photons that are emitted go undetected. But their emission is nevertheless associated with the creation of unwanted spin wave excitations in s . Suppose that the write laser was applied N times, defining N separate time bins, and that a Stokes photon was detected in the k -th time bin, leading to the creation of a spin wave excitation. When reading out this spin wave at some later time, there are several types of contributions from the above-mentioned unwanted spin waves.

(1) There are spin waves associated with Stokes photons that were emitted in the same (k -th) time bin. (a) Most of these other Stokes photons will have been emitted in other directions than the detected Stokes photon. When reading out, the corresponding anti-Stokes photons will therefore also be emitted in directions other than the anti-Stokes photon that we are interested in. They thus pose no problem for the protocol. (b) There is also the possibility of emitting more than one Stokes photon during the same time-bin in the same direction. Suppose that the solid angle that is actually detected corresponds to a fraction β of all emitted Stokes photons, and that the probability to emit a photon into this solid angle is p . In typical experiments β is in the range 10^{-4} to 10^{-5} and p in the range 10^{-2} to 10^{-3} . There is a probability p^2 to emit two photons into the same solid angle, which implies that, given the detection of a first photon, the conditional probability to have a second, undetected one (which will lead to errors in the protocol), is $2p$. The combinatorial factor of 2 arises because either of the photons in the two-photon component of the state can lead to a detection. The corresponding errors are well-known, they limit the value of p for the usual (single-mode) DLCZ protocol [3, 8].

(2) In our scenario with N time bins there are additional errors due to the undetected Stokes photon emissions in all the other time bins. When reading out the spin wave associated with the k -th time bin, the spin waves associated with all the other time bins will not be in phase, as explained above. They will thus not give rise to directional anti-Stokes emission. However, the corresponding atoms in s will nevertheless be excited to e by the read laser, the resulting anti-Stokes emission will simply be non-directional. With p and β as defined above, the mean number of atoms transferred to the state s during the write process in each time bin is $\frac{p}{\beta}$, to first order in p and summing over all directions of Stokes photon emission. (Note that typically $\frac{p}{\beta} \gg 1$ even though $p \ll 1$, cf. above). There are thus $\frac{(N-1)p}{\beta}$ atoms in s which correspond to out-of-phase spin waves. Only a fraction β of them will give rise to anti-Stokes photons that are emitted exactly into the same solid angle as the

anti-Stokes photon that we are interested in. The total error probability due to these out-of-phase spin waves is thus $(N - 1)p$.

Adding up the contributions from (1) and (2) gives a total error probability of $(N + 1)p$. There is then very little advantage from using a multi-mode protocol. This is because the choice of p in a given repeater protocol is typically determined by the size of the two-photon error. Suppose that the acceptable error is ϵ . (Its value depends on the desired final fidelity of the repeater protocol and on the number of repeater links, which depends on the distance [8].) For an N -mode protocol one then has to choose p such that $(N + 1)p = \epsilon$, or $p = \frac{\epsilon}{N+1}$. On the other hand, the repeater rate is proportional to Np for a multi-mode protocol [11], i.e. it scales like $\frac{N}{N+1}\epsilon$, which gives a modest improvement by a factor of two for large N compared to the single-mode case ($N = 1$).

We will now show that there is a way around this disappointing conclusion. We focus on (2), i.e. the anti-Stokes photons due to Stokes emissions in other time bins, which is the main error mechanism in a multi-mode protocol. The solution is to decrease the number of unwanted spin waves that are created for every detected Stokes photon, i.e. to increase the relative weight of the detected Stokes photons compared to the non-detected ones. However, this has to be done without a corresponding increase in the fraction of non-directional anti-Stokes photons that are detected, i.e. the detected fraction β has to be much greater for the Stokes than for the anti-Stokes photons, $\beta_S \gg \beta_{AS}$. Under this condition, supposing that β_S is still much smaller than one, there are now $(N - 1)\frac{p}{\beta_S}$ atoms in s , where p is still the probability to emit a Stokes photon into the detected spatial mode. Each atom in s will lead to an anti-Stokes photon emission into the detected mode with a probability β_{AS} , giving a total error $(N - 1)p\frac{\beta_{AS}}{\beta_S}$, which is much smaller than before if $\frac{\beta_{AS}}{\beta_S} \ll 1$. Adding the error $2p$ from case (1) (i.e. from the same time bin), the total error is now $p(2 + (N - 1)\frac{\beta_{AS}}{\beta_S})$. As a consequence, if the acceptable error level is again ϵ , then the multi-mode repeater rate now scales like

$$\frac{N\epsilon}{2 + (N - 1)\frac{\beta_{AS}}{\beta_S}}, \quad (1)$$

which tends towards $\epsilon\frac{\beta_S}{\beta_{AS}}$ for large N , compared to $\frac{\epsilon}{2}$ for $N = 1$. The multi-mode rate can thus be much greater than the single-mode rate in this case.

We propose to increase β_S without increasing β_{AS} by placing the atomic ensemble inside a cavity that is in resonance with the Stokes transition, but that is invisible for the anti-Stokes photons, cf. Fig. 1. Note that it is not enough for the anti-Stokes photons to be simply off-resonance with respect to the cavity, because in that case they could not leave the cavity with high probability, whereas a high collection efficiency for “good” anti-Stokes photons (those that are correlated with the

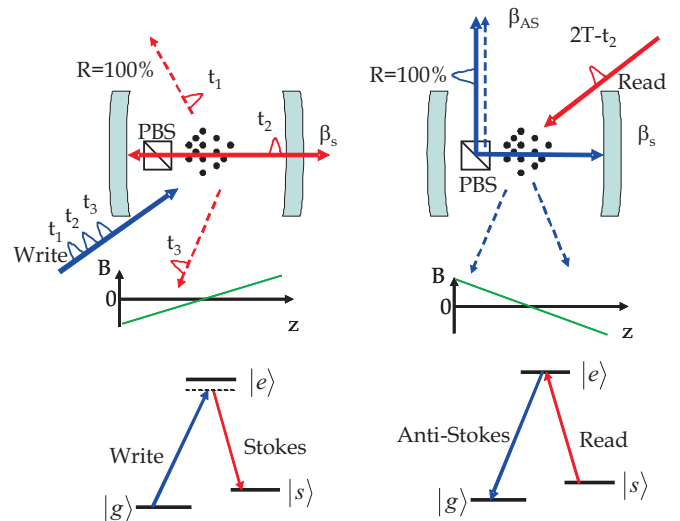


FIG. 1: Temporally multiplexed creation and recall of spin waves. Left: The atomic ensemble is excited by a sequence of write pulses. Each write pulse can lead to the emission of a Stokes photon. The cavity is in resonance with the Stokes photons, such that emission into the cavity mode is enhanced by a factor F , the cavity finesse. The cavity is asymmetric, such that the Stokes photons leave the cavity in one direction. Emission into the cavity occurs for a fraction β_S of all Stokes emissions, e.g. here at time t_2 . Every Stokes emission is associated with the creation of a spin wave, which dephases due to the applied magnetic field gradient. This dephasing can be reversed by flipping the sign of the field, e.g. at time T . The spin wave created by the Stokes emission at t_2 will be in phase at $2T - t_2$. Applying a read pulse at this time (right) creates an anti-Stokes photon, whose emission direction is correlated with that of the Stokes photon due to collective interference. A Stokes photon emitted into the cavity creates a standing spin wave, the associated anti-Stokes photon is therefore emitted into a superposition of two counter-propagating modes. The anti-Stokes photons have a polarization orthogonal to that of the Stokes photons and are ejected from the cavity on their first pass through the polarizing beam splitter (PBS). (The mirrors are highly reflective for the anti-Stokes photons.) Out-of-phase spin waves, e.g. those created at t_1 and t_3 , lead to the emission of anti-Stokes photons without any preferred direction at t_2 . Since there is no cavity-induced enhancement for them, only a small fraction $\beta_{AS} \ll \beta_S$ go in the same direction as the “good” anti-Stokes photon which is correlated to the Stokes photon from t_2 .

detected Stokes photons) is essential for a successful repeater protocol. We propose to achieve “invisibility” of the cavity for the anti-Stokes photons by having Stokes and anti-Stokes photons be at orthogonal linear polarizations [4], see Fig. 1. A cavity with finesse F enhances the emission into one of its modes by a factor of order F compared to the free-space situation, because the spectral density on resonance is increased by F [20]. The cavity concentrates the spectral density into a series of peaks of width $\frac{c}{LF}$ separated by the free spectral range $\frac{c}{L}$, where L is the length of the cavity. In the described

situation we therefore have $\frac{\beta_S}{\beta_{AS}} = F$ [21]. For example, Refs. [22] and [23] had $F = 93$ and $F = 240$ respectively for cavities containing DLCZ-type atomic ensembles.

Such moderate-finesse cavities would already allow a great enhancement in the quantum repeater rate, provided that the number of time bins N can be made sufficiently large; N is directly determined by the size of the broadening. Since the Stokes emission is an off-resonant Raman process, it can in principle be made arbitrarily fast by choosing the duration of the write pulse. However, the spin waves corresponding to different write pulses will only be fully distinguishable if there is complete dephasing of each spin wave before the next write pulse. The duration of each time bin thus has to be of order $\frac{1}{\gamma_{inh}}$, where γ_{inh} is the inhomogeneous width of the spin transition. The other factor determining N is the total time available for emission, which in the context of repeater protocols is given by the communication time L_0/c , a typical value for which is $500 \mu\text{s}$, cf. above. The total number of modes would then be of order $N \sim \frac{L_0}{c} \gamma_{inh} = 500$ per MHz bandwidth. There is no strong incentive to make N much larger than F , because the speedup thanks to multimode operation begins to saturate at that level, cf. Eq. (1). In order to fully profit from a cavity with F of order 100 a broadening of order 1 MHz is thus sufficient, with greater broadenings becoming relevant if higher-finesse cavities are used. Typical gradients used in magneto-optical traps ($\sim 10 \text{ G/cm}$) lead to Zeeman broadenings for alkali atoms of order of a few MHz for mm sized traps [24]. Spin rephasing can be induced by reversing the current direction in the coils. Storage and retrieval of light has been recently demonstrated using the reversal of a magnetic field gradient in a Rb vapor [17, 18]. In Ref. [18], a controlled broadening of 1 MHz was reversed in a few μs , which is fast enough for our purposes (the switching time should be much shorter than L_0/c).

The general requirements for implementing the DLCZ protocol efficiently apply to the present proposal as well, in particular one needs long storage times for the spin waves and a high reconversion efficiency of spin waves into anti-Stokes photons [8]. Concerning the latter, the cavity is of no assistance in the present case since it is invisible to the anti-Stokes photons. However, reconversion efficiencies as high as 50% have already been achieved for atomic ensembles in free space [25]. Long storage times can be achieved by placing the atomic ensemble into an optical lattice. Light storage for 240 ms has recently been demonstrated in a 3D lattice [26], and single spin waves have been stored for up to 8 ms in a 1D lattice [27]. In the present context, the Stokes photons couple to the cavity mode, leading to the creation of a standing spin wave which is a superposition of two plane waves with different values for $\Delta k = |\mathbf{k}_w - \mathbf{k}_S|$. The spin wave with large Δk will decay faster because it is more sensitive to atomic motion [22, 28]. In Ref. [22] the two decay times differed by two orders of magnitude. Based on Ref. [26]

this would still be compatible with ms storage times for the fast-decaying component. Ref. [29] recently stored light for over a second in a Bose-Einstein condensate.

We have proposed a way of multiplexing the DLCZ quantum repeater protocol using controlled reversible inhomogeneous spin broadening in combination with moderate-finesse cavities. Our approach, which can also be applied to improved versions of the DLCZ protocol [8, 9], opens a feasible avenue towards greatly enhanced quantum repeater performance with atomic gases.

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